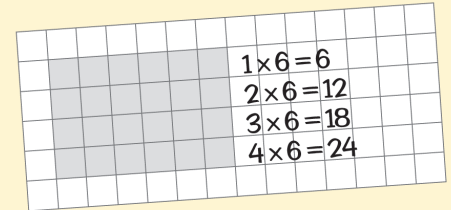


FROM MARILYN BURNS

Dear Colleague,

Students now take a break from locating products on their *Missing Products* charts and shift to a new exploration: exploring the patterns of multiples of particular factors on the multiplication chart.

Using multiples of 6 as a beginning example, students build successively larger rectangles with six squares in each row. They write a multiplication equation for each— $1 \times 6 = 6$, $2 \times 6 = 12$, and so on—continuing until they have identified all the multiples of 6 up to 144.



Then students color all of the multiples of 6 on the multiplication chart and examine the visual pattern that emerges.

X	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

In Lessons 11–15, students...

- Calculate products with factors 0 through 12.
- Represent arrangements of equal rows and rectangles with multiplication equations.
- Communicate ideas with key math vocabulary: *multiplication equation, factor, product, and multiple.*

Students repeat this exploration for multiples of other numbers and compare the patterns. In our experience, these patterns, which somewhat resemble checks and plaids, delight students, stimulate their curiosity, and motivate them to think about how the patterns and numbers connect.

These lessons strengthen students' understanding of multiplication, foster their number sense, and help build their familiarity with the products on the multiplication chart. Also included in the lessons is the game of *Pathways*, which provides students practice with the basic multiplication facts.

Marilyn Burns

“These lessons strengthen students' understanding of multiplication, foster their number sense, and help build their familiarity with the products on the multiplication chart.”

Lessons 11–15



Identify Patterns on the Multiplication Chart

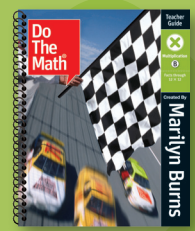
Pathways Game Board A

	32	24	15	48
28	40	35	64	20
30	12	56		16
9	25	49	42	36

③ 4 5 6 ⑦ 8

See pages 24–27 for full lesson.

See pages 28–30 for full lesson.



PLANNER

LESSON 11

LESSON 12

LESSON 13

LESSON 14

LESSON 15

<p>Lesson Summary</p>	<p>Students write products for problems in which the second factor is 6; they explore the pattern of multiples of 6 on a multiplication chart.</p>	<p>Students learn math vocabulary, color multiples of 6 on a multiplication chart, and share their observations about the patterns they observe.</p>		<p>Students learn how to play a game called <i>Pathways</i> that provides practice with multiplying the factors 3, 4, 5, 6, 7, and 8.</p>	<p>Students write equations for multiples of 4, 5, and 10, and color the products on separate multiplication charts; they share and discuss the patterns they find on the charts.</p>	<p>ASSESSMENT <input checked="" type="checkbox"/> Progress Monitoring</p> <p>Students demonstrate understanding of the objectives of Lessons 11–14 by completing <i>WorkSpace</i> pages independently.</p>
<p>Objectives</p> <p>Find an alignment to standards at www.scholastic.com/DoTheMath/community</p>	<ul style="list-style-type: none"> • Represent rectangles with multiplication equations. • Calculate products with factors 0 through 12. • Communicate ideas with key math vocabulary: <i>multiplication equation</i>, <i>factor</i>, and <i>product</i>. 	<ul style="list-style-type: none"> • Calculate products with factors 0 through 12. • Introduce key math vocabulary: <i>multiple</i>. • Communicate ideas with key math vocabulary: <i>factor</i>, <i>product</i>, and <i>multiple</i>. 		<ul style="list-style-type: none"> • Calculate products with factors 0 through 12. • Communicate ideas with key math vocabulary: <i>factor</i> and <i>product</i>. 	<ul style="list-style-type: none"> • Calculate products with factors 0 through 12. • Communicate ideas with key math vocabulary: <i>factor</i>, <i>product</i>, and <i>multiple</i>. 	<ul style="list-style-type: none"> • Calculate products with factors 0 through 12. • Communicate ideas with key math vocabulary: <i>multiplication equation</i>, <i>factor</i>, <i>product</i>, and <i>multiple</i>.
<p>Materials</p> <p>T = Teacher Bag G = Games Bag</p>	<ul style="list-style-type: none"> • <i>Multiplication Chart</i> T • <i>WorkSpace</i> pages 33 and 34 • <i>Grid Chart</i> T • <i>Do The Math Community News</i> 	<ul style="list-style-type: none"> • <i>Multiplication Chart</i> T • <i>WorkSpace</i> pages 34 and 35 • crayons or colored pencils • <i>Math Vocabulary</i> chart 		<ul style="list-style-type: none"> • <i>WorkSpace</i> pages 36 and 37 • <i>Pathways</i> Game Board A G • tiles G • dry erase markers G 	<ul style="list-style-type: none"> • <i>Multiplication Chart</i> T • <i>WorkSpace</i> pages 38–43 • crayons or colored pencils • chart paper 	<ul style="list-style-type: none"> • <i>WorkSpace</i> pages 44–48 • crayons or colored pencils
<p>Built-in Differentiation</p>	<p>The visual connection between building rectangles row by row and the way products grow focuses students on using patterns to figure products rather than just relying on recall.</p>	<p>Explicit vocabulary instruction using the <i>see it, hear it, say it, write it</i> routine helps students communicate with each other using mathematical words.</p>		<p>Playing a game with a partner provides students a “safe” way to practice the language as partners check each other’s equations and use math vocabulary to express their thinking and reasoning.</p>	<p>Observing the visual patterns of multiples on a multiplication chart builds familiarity with products and strengthens understanding of multiplication.</p>	<p>Assessing students with familiar problems allows students to show their understanding without having to approach the material in an unfamiliar format.</p>

Lessons 11–15

Identify Patterns on the Multiplication Chart

Teaching Arithmetic: Lessons for Extending Multiplication by Maryann Wickett and Marilyn Burns, pages 162, 163, 190, and 191

TeacherSpace: Multiplication CD-ROM contains videos, professional articles, and reproducibles to support teaching these lessons.

48 Identify Patterns on the Multiplication Chart

Multiplication B: Facts through 12×12
Reduced Teacher Guide, pages 48–49

LESSON 13 Learning Pathways, a multiplication game

Last Lesson Students learn math vocabulary and color multiples of 6 on a multiplication chart.

Lesson 13 Students learn a game called *Pathways* that provides practice with multiplying.

Next Lesson Students write equations for multiples of 4, 5, and 10 and color the multiples on separate multiplication charts.

Lesson Summary

Students learn how to play a game called *Pathways* that provides practice with multiplying the factors 3, 4, 5, 6, 7, and 8.

Objectives

- Calculate products with factors 0 through 12.
- Communicate ideas with key math vocabulary: *factor* and *product*.

Materials

- WorkSpace* pages 36 and 37
 - Pathways* Game Board A
 - tiles
 - dry erase markers
- = Games Bag

Language Development

Key Math Vocabulary

ENGLISH	SPANISH
factor	<i>factor</i>
product	<i>producto</i>

Academic Vocabulary

ENGLISH	SPANISH
equation	<i>ecuación</i>
game	<i>juego</i>

Cognates are shown in italics; pointing out the similarity of these words to their English equivalents will help your Spanish-speaking students acquire math vocabulary.

WHOLE GROUP

STEP 1 Teach a multiplication game.

1 Introduce the lesson.

Today, you'll learn a game called Pathways that will give you practice with multiplying.

2 Explain how to play Pathways.

There are two players in a game. The goal is to be the first player to complete a continuous pathway across the game board. Each square is a stepping-stone, and each player uses colored tiles to show his or her path.

Place a game board on a flat surface where everyone can view it.

The numbers inside the rectangle are products, and the numbers beneath the rectangle are factors.

Use a dry erase marker to circle the factors 3 and 6.

I marked these numbers because I want to multiply the factors 3 and 6. What is the product? (18)

Write on the board.

Teacher	Partner
$3 \times 6 = 18$	

If my partner agrees that 18 is the product, I place a green tile on the 18 on the game board. Then we each write the equation on our recording sheets.

Pathways Game Board A

	32	24	15	48	
28	40	35	64	20	
30	12	56	21	16	
9	25	49	42	36	
3	4	5	6	7	8

3 Demonstrate a partner's turn.

Now it's my partner's turn. He or she can erase the mark from one factor and mark another factor. Let's suppose that my partner changes the 6 to the 7.

Erase the mark from the 6, and mark the 7.

Now the marked factors are 3 and 7. What is the product of 3 times 7? (21)

I agree that 21 is correct. So my partner will put a yellow tile on the 21, and then we will each write the equation on our recording sheets.

Place a yellow tile on 21.

Write $3 \times 7 = 21$ on the board.

Pathways Game Board A

	32	24	15	48	
28	40	35	64	20	
30	12	56		16	
9	25	49	42	36	
3	4	5	6	7	8

Teacher	Partner
$3 \times 6 = 18$	$3 \times 7 = 21$

You and your partner take turns changing one factor at a time and then placing a tile on the product. The first player to make a path from one side of the board to the other is the winner.

Tell students that it is okay to mark a single factor twice. For example, when you showed changing 6 to 7, you could instead have changed 6 to 3, so the factors would be 3 and 3.

4 Demonstrate ways to complete a pathway.

Let's look at some ways that you can make a path across the board. You can make a path from top to bottom or from side to side. Any squares that share a side or a corner connect to form a path.

Take a handful of yellow markers and place them in a variety of configurations that make a continuous path. Remind students that a pathway includes squares that share sides or corners. Move your finger along the paths to show that they are continuous.

Point out that students do not have to choose products so that the tiles they place form a continuous path. They can place tiles in any squares on the board and connect them later in the game.

Pathways Game Board A

	32	24	15	48	
28		35	64	20	
30		56	21	16	
9	25		42	36	
3	4	5	6	7	8

Pathways Game Board A

18	32		15	48	
28	40		64	20	
30	12		21	16	
9	25	49	42		
3	4	5	6	7	8

Pathways Game Board A

18	32	24	15	48	
28	40	35	64	20	
	12	56	21		
9				36	
3	4	5	6	7	8

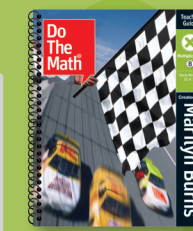
Pathways Game Board A

18	32	24	15	48	
		35			
30	12		21	16	
9	25	49	42	36	
3	4	5	6	7	8

Sample pathways

CONTINUE

Lesson 13 59



Multiplication B
 Teacher Guide
 Lesson 13

58 Identify Patterns on the Multiplication Chart

Multiplication B: Facts through 12×12
 Reduced Teacher Guide, pages 58–59

LESSON 13 continued Learning Pathways, a multiplication game

WHOLE GROUP

STEP 2 Play the game with the whole group.

1 Explain a practice turn.

Now we'll play a few practice turns. We'll play on the game board, mark the factors we pick, and place tiles on the products.

2 Take a practice turn.

I'll go first. I choose 3 and 5 as my factors. Circle the 3 and the 5.

I think the product of 3 and 5 is 15. Am I right? (yes) Always check to make sure your partner's product is correct. If you need to, you may use your multiplication chart.

I'll place a green tile on 15.

Write $3 \times 5 = 15$ on the board, and have students write the equation on *Workspace* page 36.

Teacher	Partner
$3 \times 5 = 15$	

Pathways Game Board A

18	32	24	48
28	40	35	64
30	12	56	21
9	25	49	42
3	4	5	6
7	8		

3 Students take a practice turn.

Now it's your turn. For this practice game, just tell me which factor you want to change and I'll change the factors on the game board.

Choose a student to select the new factor.

Play two or three more practice turns, choosing a different student to select the new factor for each students' turn. Write each equation on the board, and have students record the equations on *Workspace* page 36. On one of your turns, if the product is available, mark the same factor twice to remind students that this is an option.

WORKSPACE PAGE 36

Practice Recording Sheet for Pathways

DIRECTIONS

- This recording sheet is for a practice game.
- Your teacher will take the first turn.
- Record your teacher's equations in the first column.
- Record students' equations in the second column.

Teacher	Students
$3 \times 5 = 15$	

SUPPORTING INSTRUCTION

Students may have some difficulty keeping the tiles on the correct products on the game board while they are playing. You may suggest that they mark the board with Xs and Os (as in tic-tac-toe) instead of with tiles. This will eliminate any problems caused if the desk is jarred or a tile is accidentally nudged.

PARTNERS

STEP 3 Students play the game.

1 Partners play a game of Pathways.

Distribute *Game Board A*, tiles, and a marker to each pair of students.

Have students play the game and record their equations on *Workspace* page 37 as they play.

Pathways Recording Sheet

DIRECTIONS

- This recording sheet is for a game of Pathways.
- Record your partner's equations in the first column.
- Record your own equations in the second column.
- Record the date and your partner's name.

Partner	Me

SUPPORTING INSTRUCTION

As students play, check that they understand how to make a complete path. Remind them that they can go from left to right or from right to left, from top to bottom or from bottom to top, or skip around and connect the path later in the game.

2 Partners play again.

If there is enough time, have students play another game of *Pathways*, recording on blank paper.

SUPPORTING INSTRUCTION

There are five different *Pathways* game boards. The greater the factors, the higher the level of difficulty. Select easier boards for students' first games. As you progress through the module, select boards with greater factors to give students practice with more difficult multiplication facts.

HOW TO PLAY Multiplication Game

Pathways

What you need

- Pathways Game Board, tiles, dry erase marker

1

32	24	15	48
28	40	35	64
30	12	56	21
9	25	49	42
3	4	5	6
7	8		

Player A marks two factors, and places a green tile on the product.

2

32	24	15	48
28	40	35	64
30	12	56	21
9	25	49	42
3	4	5	6
7	8		

$3 \times 6 = 18$

Player B checks that the product is correct. Both players write the equation.

3

32	24	15	48
28	40	35	64
30	12	56	16
9	25	49	36
3	4	5	6
7	8		

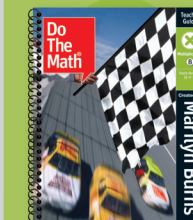
$3 \times 7 = 21$

Player B marks one new factor, and places a yellow tile on the product. Player A checks the product, and both players write the equation.

The winner is the first player to complete a path from top to bottom or from side to side of the game board.

STOP

Lesson 13 61



Multiplication B
Teacher Guide
Lesson 13 (continued)



LESSON 15 Assessing student understanding

Lesson Summary

Students demonstrate understanding of the objectives of Lessons 11–14 by completing *WorkSpace* pages independently.

Objectives

- Calculate products with factors 0 through 12.
- Communicate ideas with key math vocabulary: *multiplication equation, factor, product, and multiple.*

Materials

- *WorkSpace* pages 44–48
- crayons or colored pencils

Language Development

Key Math Vocabulary

ENGLISH	SPANISH
factor	<i>factor</i>
multiple	<i>múltiplo</i>
multiplication equation	<i>ecuación de multiplicación</i>
product	<i>producto</i>

Academic Vocabulary

ENGLISH	SPANISH
pattern	<i>patrón</i>

Cognates are shown in italics; pointing out the similarity of these words to their English equivalents will help your Spanish-speaking students acquire math vocabulary.

66 Identify Patterns on the Multiplication Chart

INDIVIDUALS

STEP 1 Students complete assessment.

1 Introduce the lesson.

Today you'll show what you know by completing WorkSpace page 44 independently. Then you'll finish coloring the multiples of 4 and 10 on the charts that you started earlier. And then you'll color multiples of one more number on another chart.

2 Students complete *WorkSpace* page 44 independently.

Explain the directions to the *WorkSpace* page. Make sure students know that when they finish the assessment they should finish coloring the charts for multiples of 4 and 10 (and the chart for multiples of 5, if they haven't finished it yet).

WORKSPACE PAGE 44

Show What You Know

DIRECTIONS
Complete.

Write equations that have a first factor of 3. Start with 3×1 and go to 3×12 .

① $3 \times 1 = 3$	② $3 \times 5 = 15$	③ $3 \times 9 = 27$
④ $3 \times 2 = 6$	⑤ $3 \times 6 = 18$	⑥ $3 \times 10 = 30$
⑦ $3 \times 3 = 9$	⑧ $3 \times 7 = 21$	⑨ $3 \times 11 = 33$
⑩ $3 \times 4 = 12$	⑪ $3 \times 8 = 24$	⑫ $3 \times 12 = 36$

Write the multiples of 6 to 72.

6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72.

Color the product of the marked factors and write the matching multiplication equation.

<p>Factors: 3 and 2</p> <p>Product: 6</p> <p>Equation: $3 \times 2 = 6$</p>	<p>Factors: 3 and 4</p> <p>Product: 12</p> <p>Equation: $3 \times 4 = 12$</p>	<p>Factors: 3 and 8</p> <p>Product: 24</p> <p>Equation: $3 \times 8 = 24$</p>
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Last Lesson Students write equations for multiples of 4, 5, and 10 and color the multiples on separate multiplication charts.

Lesson 15 Students demonstrate understanding of the objectives of Lessons 11–14.

Next Lesson Students draw squares that are based on arrangements of characters in a book.

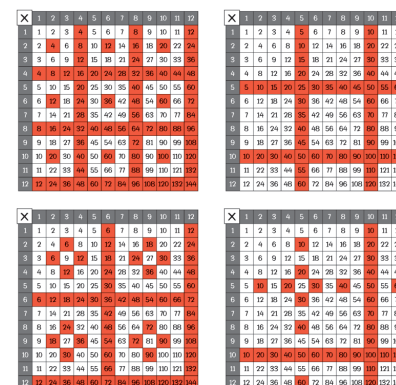
WHOLE GROUP

STEP 2 Students compare patterns on their completed charts.

1 Compare charts for multiples of 4, 5, 6, and 10.

When students have completed their charts for multiples of 4, 5, and 10, display a sample students' work of each chart along with a chart for multiples of 6. Ask students to look at the charts and talk with their partners about similarities and differences among them.

Have students share their observations about the patterns.



Charts showing multiples of 4, 5, 6, and 10

SUPPORTING INSTRUCTION

Students may need vocabulary prompts to help them describe the patterns they observe. They could use the following terms: *crosshatch, criss-cross, tic-tac-toe, plus sign, rows, columns, and plaid.* Ask questions that help students describe the patterns.

- What rows (or columns) are entirely colored in for multiples of 4? multiples of 5? multiples of 6? multiples of 10?
- Which multiple has only a tic-tac-toe pattern? (5)
- Which charts have patterns in addition to whole rows being colored? (4, 6, and 10) How would you describe the patterns? (single square, plus sign with blank center, and larger plus sign with spaces)

PARTNERS

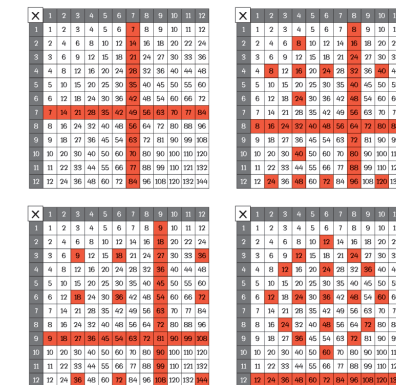
STEP 3 Students color multiples of assigned numbers on charts.

1 Students write multiples of another number and color the multiples on a chart.

Assign one of the following factors to each pair of students: 7, 8, 9, and 12. Have students complete *WorkSpace* pages 45 and 46. Partners can work together to figure out the products, but each student should record the equations and color his or her own chart.

2 Students share their results.

Choose pairs to share their charts with the class. Ask them to talk about the patterns in their charts.



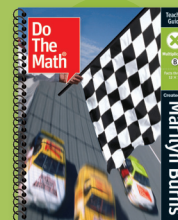
Charts showing multiples of 7, 8, 9, and 12

Have the rest of the students observe the patterns and compare them to the patterns found in previous charts.

Students may be curious about the patterns for multiples of 2, 3, and 11. Suggest that they use *WorkSpace* pages 47 and 48 to explore these multiples when they have extra time.

AFTER THE LESSON

Lesson 15 67





LESSON 15 continued Assessing student understanding

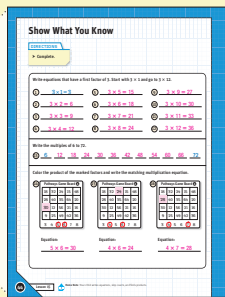
ASSESSMENT Progress Monitoring

Objectives

- Calculate products with factors 0 through 12.
- Communicate ideas with key math vocabulary: *multiplication equation, factor, product, and multiple.*

Assess

Use the annotated page to correct *WorkSpace* page 44.



Note the progress of each student in the appropriate rows on the tracking chart found on page 147 of this guide.

Differentiating Instruction

Although the lessons are carefully scaffolded and paced at a rate more likely to give students a chance for optimal learning, there will be instances when students are still struggling and need extra support. Also, there will be instances when students would benefit from additional challenges or practice. Try the teaching ideas below.

For Students Who Need More Support

- Provide grid paper for a student who needs help generating multiples of a number. Choose a number such as 9, and have the student shade 9 squares, and mark the last square. Then he or she should get to each next multiple of 9 by coloring a row of 9 more squares, or by simply adding 9.
- Play one-on-one *Pathways* with the student. This will provide you with the opportunity to help the student find and use strategies for figuring out the products.

For Students Ready for a Challenge

- Provide the student with a *Pathways* game board that has factors greater than 8 (game boards B, C, D, or E), tiles, a marker, and a sheet of paper, and have the student play the game alone.

To review the full-size Annotated Teacher Version of this *WorkSpace* see page 31.

68 Identify Patterns on the Multiplication Chart

Multiplication B: Facts through 12 × 12
Reduced Teacher Guide, page 68

Show What You Know

DIRECTIONS

Complete.

Write equations that have a first factor of 3. Start with 3×1 and go to 3×12 .

- | | | |
|---------------------|---------------------|----------------------|
| ① $3 \times 1 = 3$ | ⑤ $3 \times 5 = 15$ | ⑨ $3 \times 9 = 27$ |
| ② $3 \times 2 = 6$ | ⑥ $3 \times 6 = 18$ | ⑩ $3 \times 10 = 30$ |
| ③ $3 \times 3 = 9$ | ⑦ $3 \times 7 = 21$ | ⑪ $3 \times 11 = 33$ |
| ④ $3 \times 4 = 12$ | ⑧ $3 \times 8 = 24$ | ⑫ $3 \times 12 = 36$ |

Write the multiples of 6 to 72.

- ⑬ 6 12 18 24 30 36 42 48 54 60 66 72

Color the product of the marked factors and write the matching multiplication equation.

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28	40	35	64	20																																																																																														
30	12	56	21	16																																																																																														
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3	4	5	6	7	8																																																																																													

Equation:
 $5 \times 6 = 30$

Equation:
 $4 \times 6 = 24$

Equation:
 $4 \times 7 = 28$

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Lesson 15

Home Note: Your child writes equations, skip counts, and finds products.

Multiplication B: Facts through 12 × 12
Reduced Annotated WorkSpace, page 44

Multiplication B
Annotated WorkSpace
Lesson 15