Research Foundation Paper

GO Solve™ Word Problems: Research into Practice

Scholastic Research & Results

According to the National Council of Teachers of Mathematics, “problem solving is an integral part of all mathematics learning. In everyday life and in the workplace, being able to solve problems can lead to great advantages” (NCTM, 2000). The results of the 2003 Program for International Student Assessment (PISA), however, reveal a disappointing performance by U.S. 15-year-olds in math problem solving when compared to other leading industrialized nations. Overall, in the area of problem solving, U.S. students scored below the average score of other industrialized countries and lower than 25 of the other 38 participating countries (including non-industrialized nations).

A closer look at the results of student subgroups highlights more troubling issues for the United States. The highest-performing students from the United States “were outperformed on average by their [industrialized] counterparts,” and the U.S. had more lower-performing students than their peer countries. The U.S. performance data suggested a more pronounced relationship between socioeconomic background and problem-solving performance than most of the other participating nations. In addition, African Americans and Hispanics scored lower on average than Caucasians (NCES, 2004). When compared to classrooms in other industrialized nations, U.S. classrooms are not adequately preparing students to solve problems in a competitive global marketplace, and the gap is even larger for the neediest subgroups of students.

GO Solve Word Problems, a program developed by Tom Snyder Productions, was designed to address these issues in math problem solving. It teaches students how to better understand word problems before solving them. The program helps students see the underlying mathematical models, or situations, represented in arithmetic word problems by incorporating research-validated methods that produce good problem-solving habits and improved performance. The following paper provides an overview of the research and the *GO Solve Word Problems* approach to learning.

GO Solve™ Word Problems: Research into Practice

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GO Solve Word Problems

GO Solve Word Problems teaches students how to be better problem solvers in math. The program helps students see the underlying mathematical models, or situations, represented in arithmetic word problems. The program incorporates anchored instruction with the research-based approach known as worked examples to demonstrate and give students practice using graphic organizers to represent the information and situation in each word problem. The graphic organizers help students construct a concrete, generalizable mental model of the problem that highlights the mathematical relationships among the quantities and values. In addition, *GO Solve Word Problems* incorporates problem personalization, which has been demonstrated to improve motivation and comprehension. The design and development of the program drew on a wide range of research. The research foundation presented here links that research to the approach and implementation of *GO Solve Word Problems*.

Why We Do Not Have Good Problem Solvers

Research over the last several decades has revealed that good math problem solvers develop a representation of the problem they are attempting to solve (Riley, Greeno, & Heller, 1983; Nathan, Kintsch, & Young, 1992; Halford, 1993; English, 1997; Pape, 2004; and others). Students construct a mental model of the information and the relationships among the elements of the problem. They use this information to select a solution strategy and then apply the strategy to find the answer. The better students are at recognizing the problem situation and representing it, the better their ability to solve more complex math problems (Stigler, Fuson, Ham, & Kim, 1986; Marshall, 1995).

Unfortunately, math instruction has not consistently embraced this well-regarded approach to solving math word problems. Instead, some instruction skips the step of understanding and modeling the situation and jumps directly to the selection of a solution strategy or algorithm. For instance, focusing on key words "subverts mathematical understanding" and often leads to wrong answers (Clement & Bernhard, 2005, p. 364). The word "more" does not always suggest addition, and the word "of" does not always indicate multiplication.

Historically, the structure of the math curriculum and many math textbooks in the United States has further reinforced a procedural approach to tackling math word problems that undermines deep understanding. A textbook chapter presenting an arithmetic algorithm for students to learn is typically followed by a set of similarly structured word problems that directly reflect that algorithm, and often in a very simple form (Stigler et al., 1986). Students need only look at a few key words and numbers to apply the algorithm; they do not actually have to read and understand the problems. While the key word or direct translation approach may lead to success on these narrowly structured problems, it is not applicable to all problems. Good problem solvers tend to look beneath this surface information at the underlying problem model (Hegarty, Mayer, & Monk, 1995). "Research shows that this direct translation strategy results in a lack of conceptual understanding and the inability to transfer any problem-solving skills that are developed." (Jonassen, 2003)

It is not surprising, then, that students who have been taught with a key word or strictly procedural approach struggle with word problems that are more complex or semantically inconsistent with what they have learned (Nesher & Teubal, 1975; Fuson, Carroll, & Landis, 1996). In other words, when "more" does mean add, these students get it right. When "more" does not mean add, these students get it wrong. In addition, students who have been taught to leap to computing the answer in a word problem appear willing to accept answers that do not make sense (Verschaffel, Greer, & de Corte, 2000). Perhaps the classic example of this phenomenon is the Captain's Age Problem. French researchers presented a group of students with the following problem: "There are 26 sheep and 10 goats on a ship. How old is the captain?" Most students readily offered responses, even though the problem and their answers made no sense. Variations of this problem have been presented to students of varying ages and in different parts of the world. The responses have been consistent, leading some researchers to conclude that traditional math education has taught students to "approach word problems in a thoughtless and mechanical way" (Verschaffel et al., 2000, p. 6; see also Greer, 1997).

The GO Solve Approach

GO Solve Word Problems incorporates research-validated methods that have been shown to produce good problem-solving habits and improved performance. Specifically, *GO Solve Word Problems* explicitly introduces students to the most common types of arithmetical situations reflected in word problems. The program uses graphic organizers to help students construct concrete mental models of the situations and relationships among the information in each problem. *GO Solve Word Problems* applies a proven instructional approach built around anchored instruction and worked examples. Finally, the program enhances student motivation and text comprehension through the personalization of word problem contexts.

MATHEMATICAL SITUATIONS

For students to successfully generalize valuable problem-solving strategies, they must develop a mechanism for thinking about classes of problems rather than attacking each problem as a separate and distinct task. Researchers have typically identified this approach as "schema-based" (Riley et al., 1983; and elaborated by Marshall, 1995). For simplicity here, a schema will be defined as a framework for discriminating among and applying solution procedures to various problem situations.

For addition and subtraction, the problem situations have been well defined. Fuson acknowledges that they have slightly different names, but she succinctly identifies them as Change, Combine, and Compare (Fuson, 2003). *GO Solve Word Problems* explicitly introduces each of these situations to students (the program uses the term "Parts and Total" in lieu of "Combine"). It provides students with specific, deep examples and investigates all the possible known-unknown information combinations.

GO Solve Word Problems provides a similar presentation of the most common multiplication and division situations. While there is somewhat less consensus in this area than in addition and subtraction, researchers have consistently focused on a handful of multiplicative situations. Greer identifies three whole-number situations in particular — equal groups, multiplicative comparisons, and areas and arrays (Greer, 1992). In addition to providing direct instruction in these situations, *GO Solve Word Problems* also includes three common multiplicative situations that involve fractions and ratio relationships — part/whole, multiplicative comparison, and proportion.

GRAPHIC ORGANIZERS

Several research studies, with a range of student audiences, have demonstrated successful use of drawings or diagrams to support schema-based instruction around the situations described above. Willis and Fuson worked with students as young as second grade, employing drawings reflecting the three addition and subtraction situations. They found that students who created a correct drawing almost always selected a correct solution strategy. And the results suggested that labeled drawings would help students better identify the problem type and solve more difficult problems (Willis & Fuson, 1988; Fuson & Willis, 1989).

Buoyed by positive results among learning-disabled adults (Zawaiza & Gerber, 1993), Jitendra et al., compared schema-based instruction (using schematic organizers) to a traditional basal strategy with learning-disabled students in grades 2 through 5. The encouraging results suggested that the schema strategy could raise the achievement of learning-disabled and at-risk students to the level of their non-disabled peers. In addition, the researchers found "that students in the schema group were able to generalize strategy use to a set of novel word problems" (Jitendra et al., 1998).

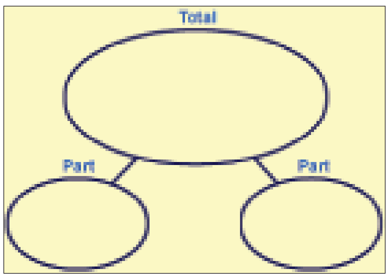
More recently, Jitendra and her colleagues have extended their research into middle school, focusing on multiplication and division situations. Again, the initial results bolster the case for using representative diagrams in a schema-based instructional model. The students in the study gained and retained word-problem-solving skills. They also were able to generalize their learning to untaught skills and new problems (Jitendra, 2002).

GO Solve Word Problems follows the model shared by these intervention studies. Like the research efforts, *GO Solve Word Problems* presents a different diagram, or graphic organizer, for each problem situation. In addition, the organizers used in the program incorporate improvements highlighted by the research, such as differentially sized boxes for additive compare problems (Willis & Fuson, 1988). These organizers (displayed on the following page) reflect the combined wisdom of complementary research efforts and were reviewed and guided by research advisors Karen Fuson and Brian Greer.

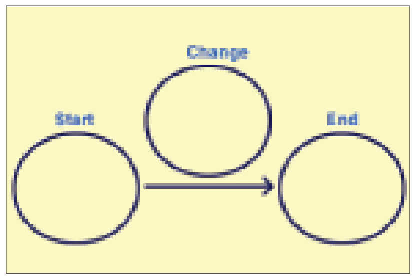
MATHEMATICAL SITUATIONS & CORRESPONDING GRAPHIC ORGANIZERS

Addition & Subtraction

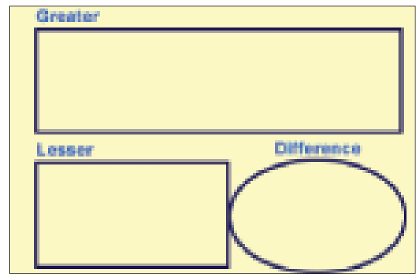
PARTS AND TOTAL



CHANGE

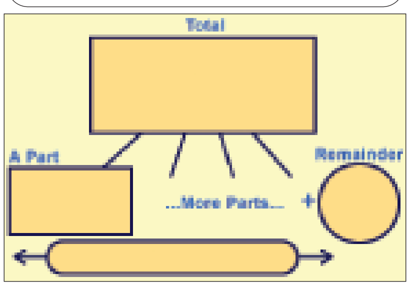


COMPARISON

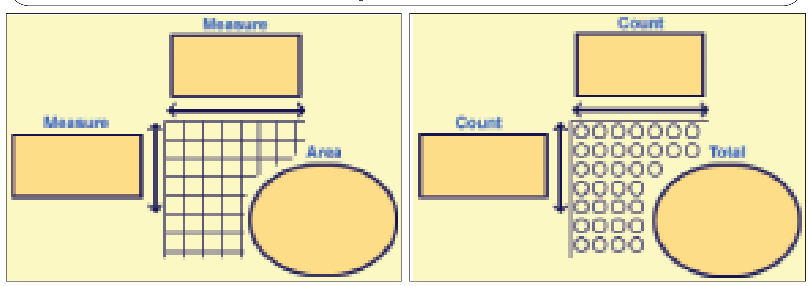


Multiplication & Division

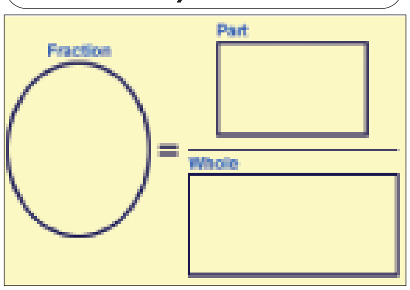
EQUAL PARTS (w/Remainders)



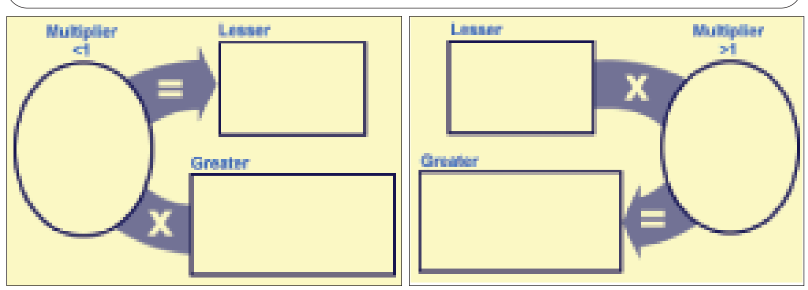
AREA/ARRAY



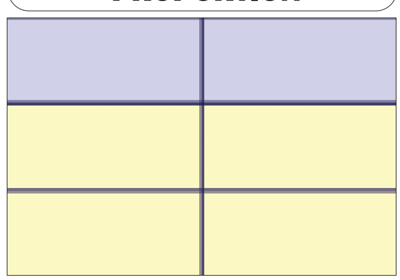
PART/WHOLE



COMPARISON



PROPORTION



ANCHORED INSTRUCTION

GO Solve Word Problems uses animated anchors to help students visualize the relationships between the graphic organizers and the actual, real world problems they represent. For example, an animated marching band facilitates students' construction of a mental model of an array situation in multiplication. A school bus that picks up students on its way to school depicts a change situation in addition. One teacher giving 3 times as much homework as another illustrates a multiplicative relationship captured by the multiplicative comparison graphic organizer. *GO Solve Word Problems* offers multiple visual examples within each mathematical situation to help students construct accurate mental models of what the words and diagrams mean.

Students need clear mental models to make sense of the problems they will encounter. And research has revealed a preference for video rather than text formats for building those models because "it is dynamic, visual, and spatial; and students can more easily form rich mental models of the problem situations" (Cognition and Technology Group at Vanderbilt, 1990). *GO Solve Word Problems* provides students with visual, concrete contexts or anchors for each mathematical situation, and the animated tutorials explicitly tie those anchors to the abstract organizers that students carry from one problem to the next.

WORKED EXAMPLES

Not only has research validated the effectiveness of using schematic graphic organizers to master word problem solving, it has also suggested a successful instructional format for teaching this approach. Cooper and Sweller, among others, have investigated the role of worked examples in schema-based problem solving (Cooper & Sweller, 1987; Sweller & Cooper, 1985). As the name suggests, worked examples involve presenting students with a thorough demonstration of working through specific problems. However, rather than presenting the solution path as a whole, worked examples break the process into clear subgoals, specifically highlighting the relationship to the problem situation, schematic organizer, and the solution strategy (for a robust summary of the research on worked examples, see Atkinson, Derry, Renkl, & Wortham, 2000). The researchers found that worked examples increased instructional efficiency and improved transfer of learning (Cooper & Sweller, 1987).

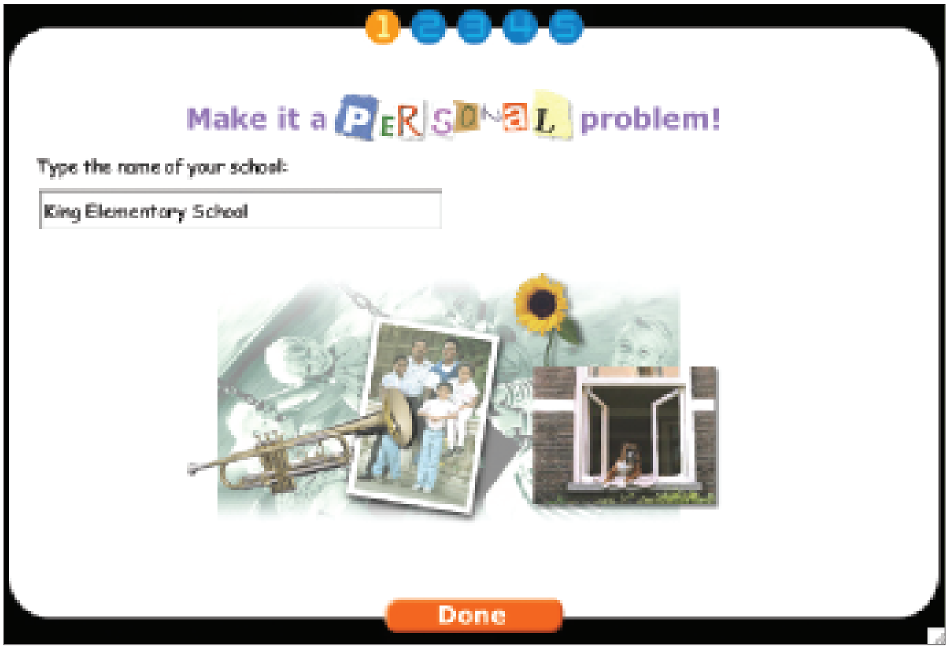
GO Solve Word Problems follows an instructional approach closely aligned to the worked examples model. The program breaks down the problem-solving process into clear subgoals for each arithmetic situation. Students first parse a mathematical situation into the organizer. The goal here is simply to recognize how the information fits into the model. Next, the program introduces an actual problem with an unknown and focuses on the variations of where that unknown can fit into the schematic organizer. *GO Solve Word Problems* then leads students through specific complexities associated with each model. After each instructional subgoal is presented, students practice applying what they have learned in the worked example. Through this systematic approach, students internalize the problem-solving schema.

For instance, for Equal Parts situations, the program focuses on variations of the number of cans of paint it takes to paint a house. Students start by placing the words in the graphic organizer — the total number of houses, the total number of cans of paint, and the number of cans of paint per house. Next, students replace the words with numbers, identifying what information is missing. The problem starts simply with 5 houses each requiring 3 cans of paint. Then the numbers get bigger, 73 large houses each needing 12 cans of paint. What if it's 6 dog houses that each only needs 1/2 of a can of paint? Each example follows the same process, regardless of the size or form of the numbers. Students compute the answers as a final step, after they demonstrate an understanding of each problem through the help of the graphic organizer. Working deeply through an example helps students gain a more thorough understanding of what they are learning.

PERSONALIZATIONS

In addition to helping students recognize and build a model of the problem situation presented in word problems, *GO Solve Word Problems* includes a research-validated technique to enhance motivation and access to problem context. Typically, word problem content has little connection to the individual student. The problems are not about people, places, or events that the learner knows or recognizes. However, computer technology allows student information to be embedded dynamically in word problems as they are presented to individual students. Research on personalized word problems has shown measurable improvements in student motivation and comprehension (Davis-Dorsey, Ross, & Morrison, 1991; Anand & Ross, 1987).

PERSONALIZED WORD PROBLEMS: measurable improvements in student motivation and comprehension



GO Solve Word Problems prompts students to personalize word problems. The program might ask the student to identify a person, such as the name of a friend, family member, or even fictional character. The program will also occasionally prompt for the name of a location or a thing, like a silly prize or the name of a roller coaster. The student's response is immediately incorporated into the next word problem — the problems become about people, places, and things familiar to students. According to the research, personalization increases student engagement and comprehension and facilitates connecting the problem to known schema.

Summary

GO Solve Word Problems draws on decades of research on word problem solving. The program integrates research-based instructional models that have been shown to produce the qualities of successful problem solvers. The program also engages students with context personalization to improve motivation and facilitate comprehension. *GO Solve Word Problems* teaches students to look beneath the surface information in word problems and recognize the underlying mathematical situations, a skill that is readily transferable to new problems and contexts.

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