Research Foundation Paper

Do The Math®

Arithmetic Intervention by Marilyn Burns
A Summary of the Research

Scholastic Research & Results

Program Overview

Do The Math is a research-based math intervention program designed to support students who are struggling with elementary arithmetic. The program was developed to address the growing national concern regarding mathematics performance in this country. The National Mathematics Advisory Panel's Final Report (2008) states that "to prepare students for algebra, the curriculum must simultaneously develop conceptual understanding, computational fluency, and problem-solving skills." With a focus on Number and Operations—the cornerstone of elementary math education and a critical foundation of algebra—*Do The Math* supports students in building a strong foundation in computation, number sense, and problem solving.

The National Mathematics Advisory Panel report further states that "by the end of Grade 5, students should be fluent with whole numbers and by the end of Grade 6 they should be fluent with fractions." *Do The Math* addresses these exact topics by focusing intervention on addition, subtraction, multiplication, division, and fractions in twelve carefully sequenced modules. Each module consists of thirty 30-minute step-by-step lessons that are scaffolded and paced for students who struggle with math.

Do The Math is the result of the collaborative work of a research and development team headed by Marilyn Burns and contributed to by *Math Solutions Professional Development* master teachers. Marilyn Burns has worked with students and teachers and continues to teach regularly so as to deepen her understanding and insight into the needs of struggling students and the teachers who teach them.

This Research Foundation Paper provides a brief review of the learning challenges facing math students in the United States and follows with the eight guiding principles that drove the development of *Do The Math*. The supporting research literature and an example from the program are provided for each guiding principle.

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Introduction

According to the 2007 National Assessment of Education Progress (NAEP) Mathematics test, 61 percent of American's fourth graders are not proficient in mathematics. The NAEP data also reveals that 68 percent of eighth graders are not proficient in mathematics. More students in the U.S. need to be proficient in mathematics in order to be successful in algebra. The National Mathematics Advisory Panel's Final Report (2008) establishes fluency with fractions and other basic arithmetic concepts and skills as critical foundations for algebra.

One percent of school-age children experience a math disability not associated with any other learning disability, and two to seven percent experience serious math deficits. Students with mild disabilities do not perform as well as their peers without disabilities on basic operations, and this discrepancy in performance increases with age (Cawley, Parmar, Yan, & Miller, 1996). In addition, students with math disabilities may respond with lower self-esteem, avoidance behaviors, and decreased effort. Learning math is also a challenge for many English language learners, as the content presents its own unique academic vocabulary and is often presented abstractly.

The *No Child Left Behind* act requires that all students reach proficiency in math by 2014, and the National Council of Teachers of Mathematics (NCTM) goals (2000) aspire for all students to become mathematical problem solvers, learn to communicate and reason mathematically, use representations to model problem situations, and make connections among mathematical ideas. In addition, the National Mathematics Advisory Panel recommends that math curricula for elementary and middle school be a coherent progression of key topics with an emphasis on proficiency. For many students, especially those who struggle, meeting these goals presents a challenge when they only receive the typical 50 minutes a day dedicated to math instruction. Moreover, many students require instruction that is specifically designed to meet them at their level and to focus on the most critical foundational mathematical concepts.

Do The Math addresses these learning challenges facing American students. The program's instructional design applies what is known about reaching a wide variety of students who struggle with math to achieve proficiency with arithmetic concepts and skills, by incorporating the following guiding principles:

- Scaffolded Content
- Explicit Instruction
- Multiple Strategies
- Gradual Release
- Student Interaction
- Meaningful Practice
- Assessment & Differentiation
- Vocabulary & Language

To see further evidence of how *Do The Math* aligns with the National Mathematics Advisory Panel's Final Report, refer to the related citations throughout this paper.

Scaffolded Content

Guiding Principle

Scaffolding is the systematic process of analyzing the content and partitioning it into small manageable chunks for the purpose of planning and delivering instruction that facilitates students' learning.

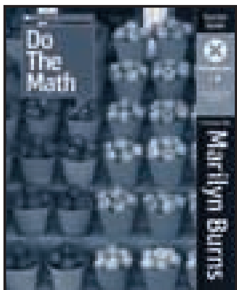
Scaffolding calls for identifying and sequencing the concepts and skills that are essential to the content being taught. Once the content is scaffolded, instruction can then be organized in a way that supports students' learning and paced to allow students sufficient time and practice to be successful. Research shows that scaffolding content to inform instruction benefits all students, and particularly students with learning disabilities.

Research Foundations

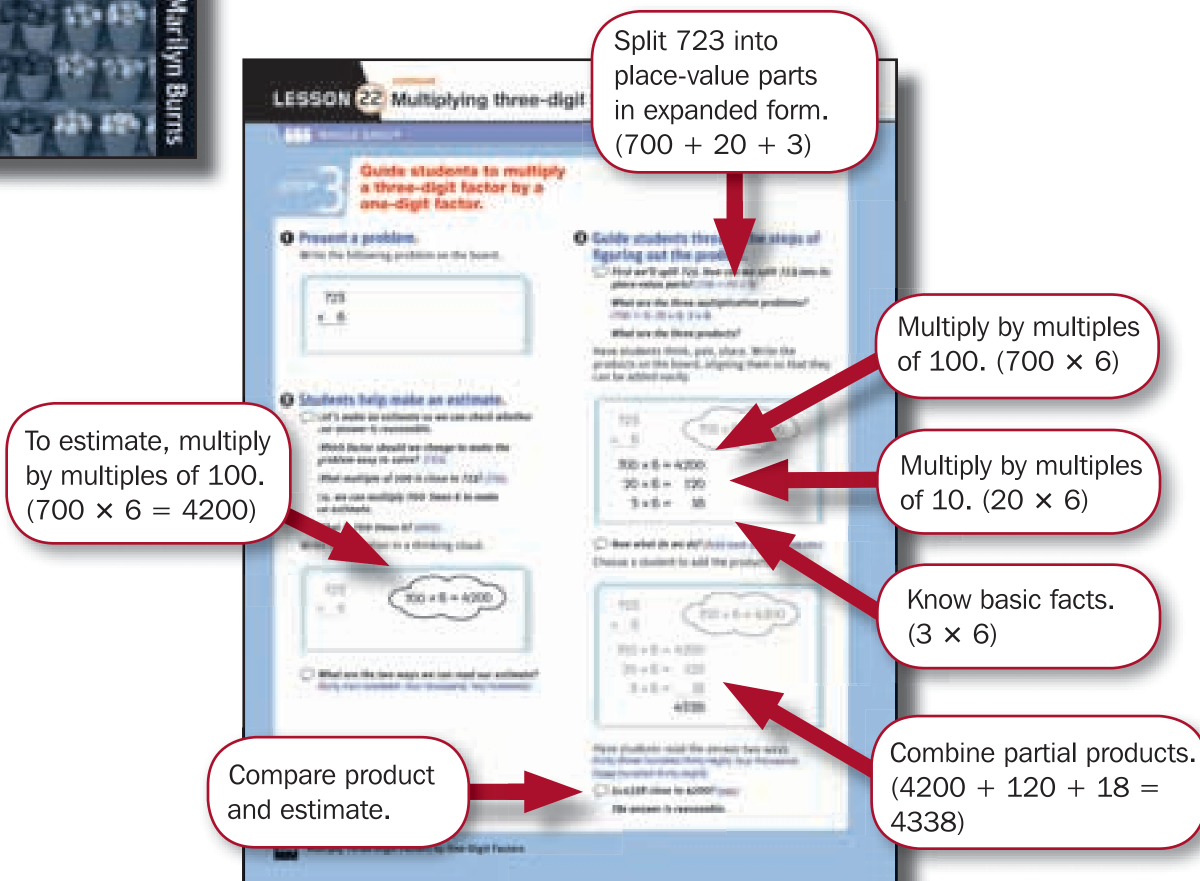
- A synthesis of investigations into instructional techniques for students with learning disabilities shows scaffolding to be among the most effective approaches (Gersten, 1998).
- Three strategies for scaffolding content—organization of concepts, sequencing, and chunking—support teaching for conceptual understanding (Grouws & Cebulla, 2000).
- Students learn better when new knowledge is connected to things that they already know and understand (Hiebert & Carpenter, 1992; Hiebert et al., 1997).
- Scaffolding is one of the principles of effective instruction that enables teachers to accommodate individual student needs (Kame'enui, Carnine, Dixon, Simmons & Coyne, 2002).

Example From *Do The Math*

Do The Math focuses on the basics of Number and Operations with lessons that build accuracy, efficiency, and understanding. All lessons have been carefully designed and sequenced to align with the scaffolding of the content, and then paced to ensure student success.



In *Multiplication C: Lesson 22*, in order to multiply 723 x 6, students must have the necessary prerequisite foundation of skills and concepts.



In this module, content is **scaffolded** and lessons are **sequenced** to provide all of the skills necessary to be successful with this problem.

Explicit Instruction

Guiding Principle

Explicit instruction is a strategy in which the teacher: 1) demonstrates and provides clear models of how to solve a problem or learn a skill, 2) guides students to understand and articulate relationships, 3) provides extensive practice with timely feedback, 4) encourages students to verbalize their thinking, and 5) helps students make connections between their mathematical experiences and the concepts and skills.

Explicit instruction has consistently resulted in positive effects on the performance of students who have difficulty with mathematics. Students who need intervention instruction typically fail to look for relationships or make connections among mathematical ideas on their own. Explicit instruction provides the help needed to connect new learning to what students already know and then be given the opportunities to apply new concepts and skills to relevant problems. The routines and consistent language used in explicit instruction also provide English language learners the clear, specific, and easy-to-follow steps they need as they learn a new skill or strategy.

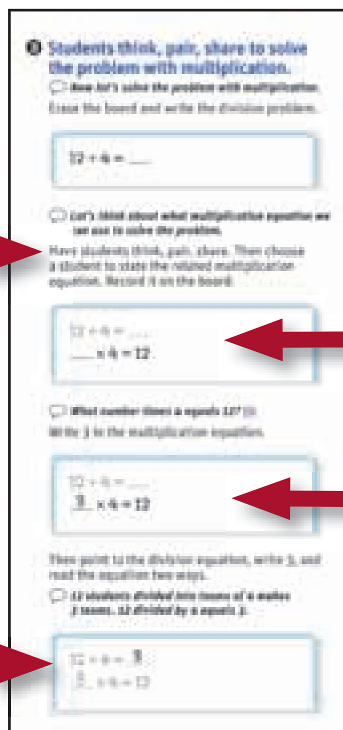
Research Foundations

- Explicit instruction with students who have mathematical difficulties has shown consistently positive effects on performance with word problems and computations (National Mathematics Advisory Panel, 2008).
- A meta-analysis of research shows that substantial evidence supports the effectiveness of explicit instruction (Adams & Engelmann, 1996).
- Reports from the National Follow-Through Project showed that explicit instruction was associated with achievement in both basic skills and math concepts (Hall, 2002).
- Teaching underlying mathematical foundations through explicit instruction and then providing students with the opportunity to work on relevant problems produced positive gains when compared with traditional instruction (Gersten, 2003).
- Explicit concrete-to-representational-to-abstract sequenced instruction was shown to be an effective pedagogical strategy for building mathematical knowledge and skills (Witzel, Mercer, & Miller, 2003).
- A meta-analysis of fifty studies shows that systematic and explicit instruction had a strong positive effect for both special education and low-achieving students (National Council of Teachers of Mathematics, 2007).

EXAMPLE FROM *DO THE MATH*

In *Do The Math* explicit instruction utilizes the scaffolded content and is designed to guide teachers to model, connect concepts to their mathematical representations, and introduce appropriate language. In *Do The Math*, explicit instruction does not mean to imply "teaching by telling." When learning requires understanding of logical mathematical processes, it is essential that the explicit instruction presents carefully sequenced experiences through which the students develop concepts, learn skills, see relationships, and make connections. However, when students are required to learn social conventions, such as vocabulary and mathematical symbols, the explicit instruction imparts the necessary and appropriate information.

In *Division B: Lesson 2,* the teacher uses the problem 12 ÷ 4 to explicitly reveal the relationship between division problems and multiplication problems.



Students **think**, **pair**, **share** and then state the related multiplication problem.

The teacher reminds students of the connection between division and multiplication.

The teacher writes the symbolic representation of the related multiplication problem on the board.

Students solve the multiplication problem and the teacher points out how the 3 in the multiplication problem relates to the 3 in the division problem.

The teacher **explicitly** makes the connection between multiplication and division.

Multiple Strategies

Guiding Principle

Using a range of teaching strategies and contexts to teach concepts and skills helps ensure that all students learn and make connections.

Approaching mathematical knowledge through the use of modeling with manipulatives, interacting with mathematical ideas through literature, engaging in discussion of math ideas and skills through games, and viewing and creating visual representations gives the best possible chance for all students to build number sense, develop skills, and deepen their mathematical understanding.

Research Foundations

- Studies that included visual representations along with other components of explicit instruction produced significant positive effects for students with learning disabilities and low-achieving students (National Mathematics Advisory Panel, 2008).
- Instruction that makes use of multiple representations for conveying mathematical concepts allows a greater proportion of students to access ideas and deepen understanding, as compared with instruction that relies on a single representation (Ozgun-Koca, 1998; Goldin, 2000; McArthur et al., 1988; Yerushalmy, 1991).
- Because instructional strategies reach students in different ways, the use of multiple strategies provides enhanced opportunities to reach more students and to strengthen deep conceptual understanding for all learners (Tomlinson, 1999).
- A meta-analysis of research confirms that physical manipulatives can support learning (Sowell, 1989).
- A concept paper published by the American Mathematical Society has been influential in identifying some common areas of agreement, one of which is that mathematics should be taught using multiple strategies (Ball et al., 2005).
- A summary of research suggests that the development of practical meaning for mathematical concepts is enhanced through the use of manipulatives (Sabean & Bavaria, 2005).

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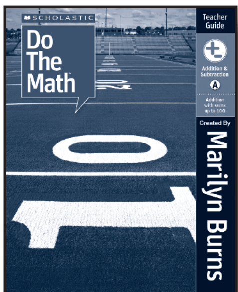
Do The Math
Created by Marilyn Burns

Example From *Do The Math*

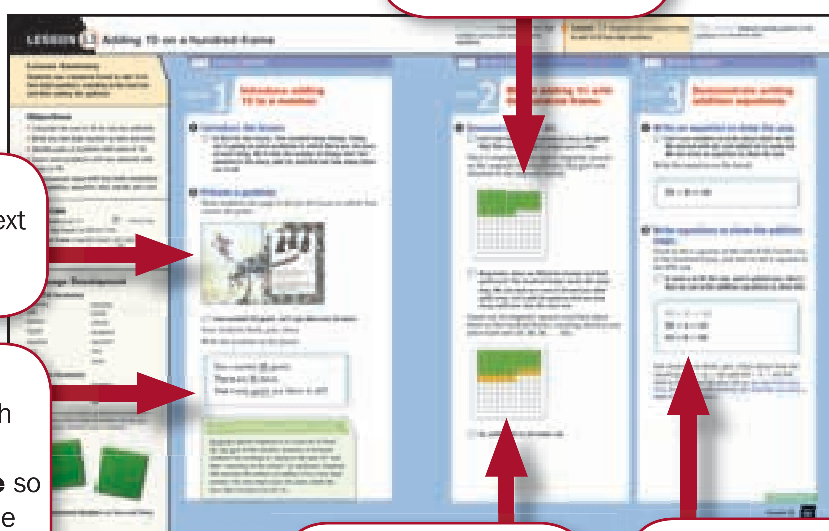
In *Do The Math,* lessons engage students with concepts and skills in multiple ways using concrete manipulative materials, games that reinforce and provide practice, selected children's literature that provides a context for mathematical concepts and skills, and visual representations to help students represent their thinking.



In Addition & Subtraction A: Lesson 13, a children's book provides students a context both for representing different quantities on the hundred-frame with tens and ones and also for adding 10 to any number.

Manipulatives: The teacher models the problem with a hundred frame using magnetic strips and squares.

Literature is used as a context for adding 10 to a number.



The problem is presented with words using a **problem-frame** so students become familiar with the format.

The **hundred frame** provides a clear **visual representation** of tens and ones.

Abstract representation: The teacher represents the solution with equations.

In this lesson **multiple strategies** including literature, problem frames, and manipulatives are all used to teach students how to add 10 to a number.

Gradual Release

Guiding Principle

In Gradual Release pedagogy, the focus of instruction is on the level of responsibility that the teacher maintains to ensure that students understand and can complete a particular task on their own.

The gradual release process begins with modeling new content, followed by guided practice in which students take on increasing cognitive responsibility. This is followed by paired practice, giving students the opportunity to talk to each other about their reasoning to solve a problem, and finally students are released to work independently. This process of moving through phases from dependence to independence has been shown to be an effective strategy, ensuring optimal learning and achievement.

Research Foundations

- Guided practice, in which students take on increasing cognitive responsibility, is an effective strategy across a wide variety of settings and contexts (Pearson & Gallagher, 1983).
- When students first learn a new concept or skill, the teacher carries most of the cognitive weight, providing extensive modeling, and articulating strategies and thought processes for students. This type of support is essential for bridging the gap between what students know and can do on their own and the knowledge and skills they need to move to the next phase of learning (Rose, 2004).
- Optimal learning is achieved when students move through phases of dependence to independence through the guidance of a teacher using a gradual release of responsibility model of instruction. The report on the research defined four phases of learning: demonstration, shared demonstration, guided practice, and independent practice (Routman, 2003).

Example From *Do The Math*

In *Do The Math*, gradual release pedagogy sets an expectation for student involvement and gives learners the direction and the support needed to be successful. It consists of four phases.

- In **Phase 1**, the teacher models and records the appropriate mathematical representation on the board.
- In **Phase 2**, the teacher models again but this time elicits responses from the students.
- During **Phase 3**, the teacher presents a similar problem. Students work in pairs to solve the problem. The teacher records their solution on the board.
- Finally, in **Phase 4**, students work independently, referring to the work recorded on the board if needed.

In *Fractions B: Lesson 11*, students learn to compare fractions one unit from 1 whole as the teacher uses the gradual release model.

Phase 1

The teacher introduces the *compares fractions one unit from 1 whole* strategy and **models** how to use the strategy to compare $\frac{7}{8}$ and $\frac{5}{6}$.

Phase 2

The teacher elicits responses as she **guides** students to compare fractions using the strategy and verifying with manipulatives.

Phase 3

Student **pairs** explain how to compare $\frac{2}{3}$ and $\frac{3}{4}$ using the strategy.

Phase 4

Students work **independently** to compare fractions such as $\frac{3}{4}$ and $\frac{7}{8}$ using this new strategy.

This lesson demonstrates the **gradual release** process from teacher demonstration to students comparing fractions on their own.

Student Interaction

Guiding Principle

When students voice their mathematical ideas and explain them to others, they extend and deepen their understanding of the mathematics. Interactions help students make sense of what they are doing and help them to clarify, explain, and evaluate their own thinking and the thinking of their partners.

Pairing students to interact with each other encourages each of them to take responsibility for their own learning as they discuss their thinking when they disagree or do not understand their partner's reasoning. Student interaction can occur in whole groups, small groups, or with pairs of students solving a problem together, playing a game, practicing how to explain how they solved a problem, or reporting to the class how they solved a problem. The opportunity for students to express their math knowledge verbally to a partner is particularly valuable for many students who are developing English language skills.

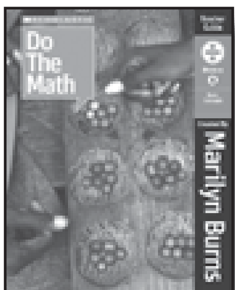
Research Foundations

- Several studies show that collaborative learning methods such as peer-mediated instruction produces increased achievement and conceptual understanding for students with and without disabilities (Fuchs et al., 1997).
- Learning with understanding can be further enhanced by classroom interactions, as students propose mathematical ideas and conjectures, learn to evaluate their own thinking and that of others, and develop mathematical reasoning skills (Hanna & Yackel, 2000).
- Classroom discourse and social interaction can be used to promote the recognition of connections among ideas and the reorganization of knowledge (Lampert, 1986).
- Cooperative learning enhances students' enthusiasm for learning and their determination to achieve academic success and has been shown to increase the academic achievement of students of all ability levels (Lan & Repman, 1995; Stevens & Slavin, 1995a, 1995b).
- When students explain their thinking, either in pairs or small groups, they are forced to reflect on their own reasoning and to organize their thoughts clearly in order to communicate them to others (Chapin et al., 2003).
- Structuring instruction around carefully chosen problems, allowing students to interact when solving problems, and then providing opportunities for them to share their solution methods result in increased achievement on problem-solving measures (Grouws & Cebulla, 2000).
- The benefits of cooperative learning include not only increased knowledge and skills, but also increased conceptual understanding, improved attitudes or motivation, improved communication skills, and improved social skills (Davidson, 1990).

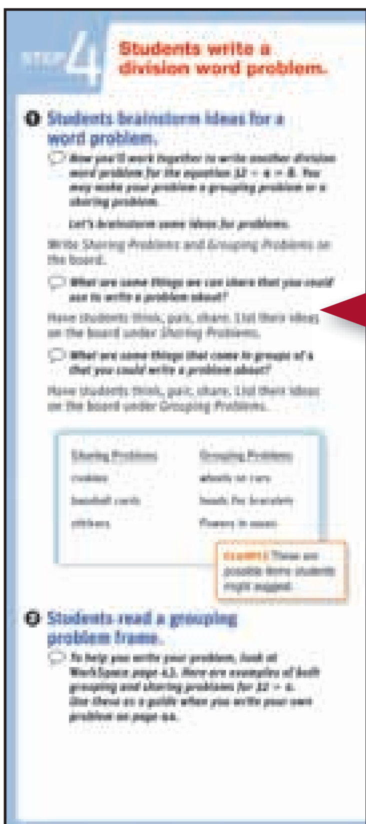
EXAMPLE FROM *DO THE MATH*

In *Do The Math* student interaction is built into the program.

- One essential routine that encourages active student engagement is **think, pair, share**. Having students talk in pairs provides them a safe way to share ideas, brainstorm, and practice what they will say when they share with the larger group.
- Partner interaction is always encouraged when students are released to work independently in their *WorkSpace* books.
- Games encourage active engagement and provide practice.



In *Division A: Lesson 26*, students **think, pair, share** ideas for writing sharing and grouping word problems based on 32 ÷ 4 = 8. After **conferring** they offer their ideas and the teacher records them on the board. When students work independently they write their own word problems—one example of a grouping problem and one example of a sharing problem.


STEP 4 Students write a division word problem.

1 Students brainstorm ideas for a word problem.

2 Students read a grouping problem frame.

In this lesson, students **think**, **pair**, **share** to come up with problem ideas for 32 ÷ 4 = 8.

Students **interact** in order to think of grouping or sharing word problems.

Meaningful Practice

Guiding Principle

Meaningful practice is practice that is based on conceptual understanding, number sense, and connected to previously learned concepts and skills.

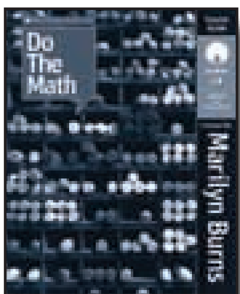
Practice helps strengthen and reinforce what has been taught and learned. Practice that is meaningful is based on number sense and understanding rather than learning and practicing rote procedures. Mathematics makes more sense and is easier to remember when students connect the new knowledge to existing knowledge and solve problems in ways based on understanding of Number and Operations. Meaningful practice provides students opportunities to strengthen and reinforce their learning and maximize their success.

Research Foundations

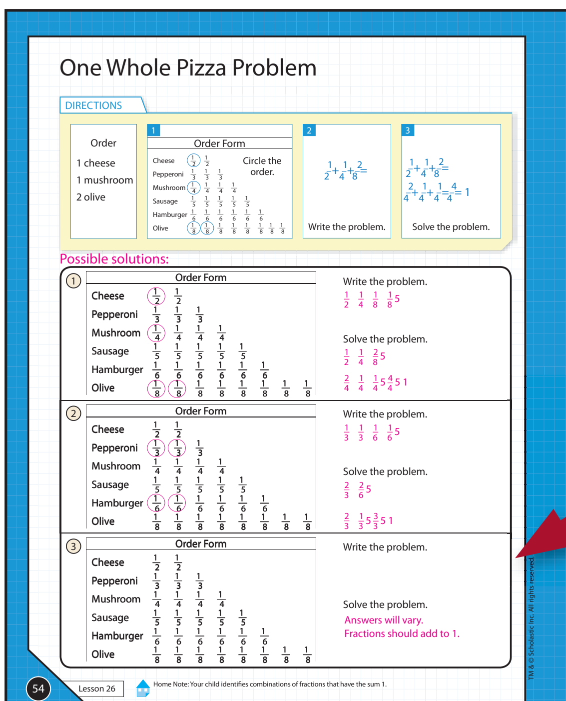
- A review of 26 high-quality studies indicates that explicit instruction with extensive practice is effective for students with learning disabilities and for low-achieving students (National Mathematics Advisory Panel, 2008).
- In a report to the U.S. Department of Education, two specific strategies were cited that help students gain a deeper understanding of a topic. One strategy is targeted assignments that focus on specific elements of a complex skill and practice. The other is practice that focuses on building conceptual understanding related to skills and procedures (Marzano et al., 2000).
- An important reason to practice mathematics is to strengthen and reinforce what has already been learned. Studies about the specific nature of practice suggest that many repeated practice sessions are required for students to attain high levels of competence (Newell & Rosenbloom, 1981).
- Repeated practice and application are essential to learning (Gee, 2003; Marzano, 2002; Pressley, 1995).
- Research has shown irrelevant text elements are often distracting to students with special needs (Seidenberg, 1989).
- When students understand what is expected of them they have increased motivation (Reiser & Dick, 1996).
- Mathematics makes more sense and is easier to remember and to apply when students connect new knowledge to existing knowledge in meaningful ways (Schoenfeld, 1988).

Example From *Do The Math*

In *Do The Math,* practice is an essential part of every lesson. The written practice in the *WorkSpace* is always similar to what students experienced during the lesson. The practice has been carefully sequenced so that no new knowledge or skill is required in order for the student to be successful. Practice is supported through visual directions on the *WorkSpace* pages for those students who need a point-of-use reminder of the steps involved.



In *Fractions C: Lesson 26*, students practice figuring out combinations of fractions that have a sum of 1 through the context of ordering pizza by the slice. All of the prerequisite skills—generating equivalent fractions, adding fractions both mentally and with paper and pencil, and simplifying fractions—have been learned and practiced previously. Now students use their understanding and number sense about fractions to practice solving contextualized problems.



This *WorkSpace* page contains practice problems that arose from the literature experience in earlier lessons, thus making the problems interesting, connected, and motivating.

The **meaningful practice** strengthens students' understanding, is motivating, and is accessible so that students are successful.

Assessment & Differentiation

Guiding Principle

Differentiation is an instructional approach based on the principle of equity—that all students, regardless of their personal characteristics, backgrounds, physical challenges, language challenges, and learning challenges, must have opportunities and support to learn.

Differentiation of instruction is essential in order to meet the needs of all students and is a significant challenge for classroom teachers. However, there are some students for whom the differentiation provided during regular math class are not sufficient for them to be successful. Formative assessments are key for identifying these students and their needs. Then, intervention is required to provide these students instructional support in addition to their regular classroom instruction. During intervention instruction, students will progress at different rates and sometimes need additional support. For this reason, an intervention program must also include specific suggestions for differentiating instruction to provide for the success of all students.

Research Foundations

- Findings from a review of the high-quality studies of assessment suggest that use of formative assessment benefited students at all ability levels. When teachers use the assessment data to provide differentiated instruction, the combined effect is significant (National Mathematics Advisory Panel, 2008).
- Well-documented examples demonstrate that all children, including those who have been traditionally underserved, can learn mathematics when they have access to high-quality instructional programs that support their learning (Campbell, 1994; Griffin, Case, & Siegler, 1994; Knapp et al., 1995; Silver & Stein, 1996).
- Grouping students for instruction and engaging learners, both practices central to differentiation, have been validated as effective (Ellis & Worthington, 1994).
- Providing teachers with specific information on how each student is performing consistently enhances students' mathematics achievement (Baker et al., 2002).
- Research has shown that building upon students' prior knowledge and directly addressing misconceptions can lead to increased learning (Swan, 2002; Askew, 2002).
- The study of a professional development model that helps teachers to understand their students' thinking revealed that when teachers changed their instruction to incorporate more problem-solving activities and to consider students' varied solution strategies and thought processes, student achievement gains increased accordingly (Fennema et al., 1996).

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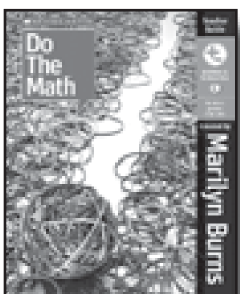
Example From *Do The Math*

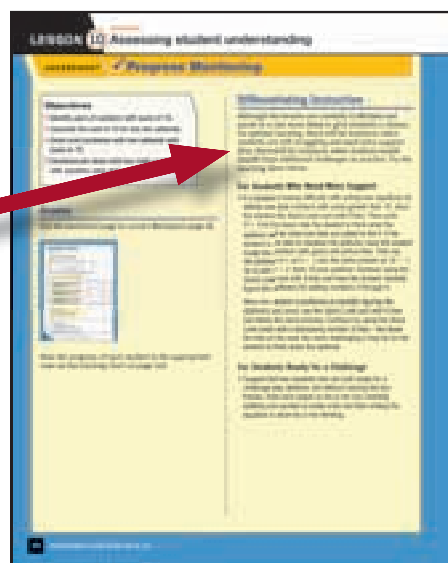
In *Do The Math* lessons are carefully built on scaffolded content with attention to the common misconceptions of students who are in need of intervention. Ongoing assessment and suggestions for differentiation are integral to the program.

- The **Beginning-of-Module Assessment** establishes a benchmark with which to measure each student's mathematics growth after completing the module.
- **Formative Assessment** through daily observations gives students the prompt attention that will enable them to complete math assignments successfully.
- **Progress Monitoring,** which occurs every fifth lesson, is followed by suggestions for differentiating instruction—what to do for the students who need additional support and those ready for a challenge.
- The **End-of-Module Assessment,** or **Summative Assessment,** provides an opportunity to measure student growth and an opportunity to give continued support to those who need it.



In *Addition & Subtraction C:* Lessons 6–10, the teacher **observes** students and, if needed, helps them understand that 14 tens equals 140, and in Lesson 10 **monitors their progress** through a **written assessment**, then provides **additional support** or challenges to meet the needs of individual students.

Following Lesson 10, the teacher is provided with suggestions for differentiating instruction with additional support for students who need it and ways to extend the learning for students who are ready for a challenge.



Ongoing assessment helps teachers identify and differentiate instruction to meet every student's needs.

Vocabulary & Language

Guiding Principle

Teaching students correct mathematical language gives them the tools to articulate their mathematical thinking coherently and precisely. Research shows that explicit instruction in mathematics vocabulary supports success with math problem solving.

While many of the words that are used to describe mathematical ideas are familiar to students, their meanings in general usage are often very different from their mathematical meanings. Mathematical vocabulary is determined by social convention, in contrast to the logical foundations of mathematical ideas, which call for thinking and reasoning. This distinction is key to making instructional decisions. Because there is no way to figure out, for example, that numbers divisible by 2 are called ***even numbers***, the best pedagogical choice is to provide that information to the student, that is, to teach by telling.

Explicit vocabulary instruction introduced after students develop conceptual understanding of a mathematical topic, idea, or property makes the vocabulary more meaningful to the student as it connects the new word or words to the mathematics that the student has already experienced.

Students incorporate the new vocabulary into their own language as they explain their thinking to each other or to the whole group when they hear the word or words used consistently and regularly by the teacher and other students.

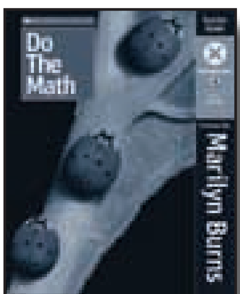
Explicit vocabulary instruction not only benefits native English speaking students, but is particularly helpful to English language learners. Access to appropriate and effective instruction of academic vocabulary supports English language learners' second language development and facilitates their understanding of the math instruction. Explicitly teaching vocabulary and then using the words frequently in class discussions benefits all learners and encourages them to use the words when they are explaining their reasoning to each other and to the larger group as well.

Research Foundations

- A cause of confusion in mathematics for students is that many mathematical terms are also used in everyday language and the common-usage meanings are often quite different from the mathematical ones (Shuard & Rothery, 1984).
- Based on a study of mathematical discourse between teachers and students in math classrooms, it was demonstrated that confusion arises when the precise meaning of mathematical words is not established (Raiker, 2002).
- Instruction that emphasizes vocabulary and domain-specific communication skills can support learning for all students (Allen, 1988; Ball et al., 2005).
- Systematic vocabulary instruction in which new vocabulary is directly defined increases the likelihood that students will learn the terms (Marzano, 2002).
- Students learning English as an additional language (EAL) who are struggling with math must overcome confusions between trying to achieve mathematical understanding and trying to learn mathematical procedures (Frederickson & Cline, 2002).
- Explicit vocabulary instruction is important because students may have existing notions about words such as *product, factor, times,* and *sum* that do not align with the mathematical meaning of these terms (Allen, 1988; Ball et al., 2005).
- Student understanding of new concepts may be enhanced through instruction that uses routines, embeds redundancy in lessons, provides explicit discussion of vocabulary and structure, and teaches students metacognitive skills (August & Hakuta, 1997).

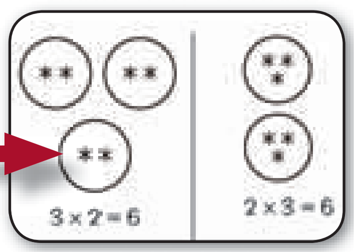
Example From *Do The Math*

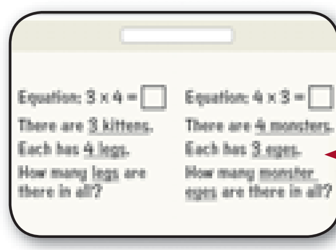
In *Do The Math*, vocabulary is introduced after students experience and develop a firm understanding of the mathematical concept so that they can anchor the word in their understanding. The meaning of a key vocabulary word is explicitly taught using the routine of **see it, hear it, say it, write it, and read it**. The word is recorded on a math vocabulary chart with examples so that students may refer to it as needed. Students read the meaning in their own student glossaries and record the meaning with an example in their *WorkSpace* books. Students hear the word used frequently by the teacher and naturally begin to incorporate it into their own explanations as they talk to their partners and share their reasoning with the whole group.



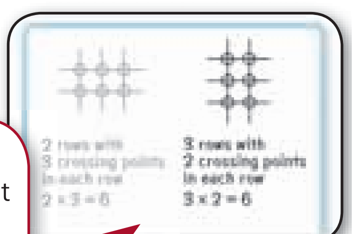
In *Multiplication A*, students experience the commutativity of factors when they write equations for equal groups, and they experience it again when writing and solving problems. They understand that 2 × 6 and 6 × 2 have the same product, but the word problems that could be written for each of them are quite different. Once they've had these experiences, the actual vocabulary—*Commutative Property of Multiplication*—is introduced using the **see it, hear it, say it, write it, and read it** routine.

Students first experience the concept of commutativity as they play *Circles and Stars*. 3 groups of 2 stars has the same total as 2 groups of 3 stars.

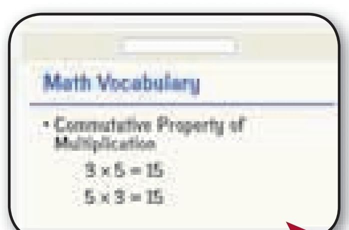




Students experience commutativity again with word problems. In this example they find that both 3 × 4 and 4 × 3 equal 12.



Students learn that the position of the arrangement of rows does not affect the product. 3 rows of 2 rotated becomes 2 rows of 3 and the number of crossing points or product remains the same—6.



Finally, students learn that this concept has a name—it is called the *Commutative Property of Multiplication.*

Students experience the concept in many lessons before the actual **vocabulary** is taught using the **see it, hear it, say it, write it, and read it** routine.

About Marilyn Burns

Marilyn Burns is one of today's most highly respected mathematics educators. Over the course of more than 40 years, Marilyn has taught children, led in-service sessions, and written a variety of professional development publications for teachers and administrators.

In 1984, Marilyn formed **Math Solutions Professional Development**, an organization dedicated to the improvement of math instruction in grades K–8. Soon after, Marilyn began writing and publishing to further support teachers and provide districts with the resources they needed to implement in-depth and long-lasting change in their schools. Her most recent contributions include the math, literature, and nonfiction series and the *Lessons for Algebraic Thinking* series.

In 1996, Marilyn Burns received the Glenn Gilbert National Leadership Award from the National Council of Supervisors of Mathematics for her influence on mathematics education. The nominators took special note of Marilyn's humor and compassion, saying:

> **"Few professionals have touched and inspired so many math educators. She has taught us several important lessons . . . We must treat teachers with respect, honesty, and a thoughtful vision. We must turn to student work to make sense of student understanding and achievement."**

In 1997, Marilyn received the Louise Hay Award for Contributions to Mathematics Education from the Association for Women in Mathematics.

In collaboration between Scholastic and Math Solutions Professional Development, Marilyn created *Do The Math*, a 12-module intervention program that focuses on Number and Operations. It was released in 2008 and specifically targets addition, subtraction, multiplication, division, and fractions.

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